

Sectional Properties of Reinforced Concrete Sections

Notes for Using Inertia-RC

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Introduction

Inertia-RC is a tool for the analysis and design of Reinforced Concrete Sections in the elastic range.

The properties of reinforced concrete sections are required when estimating deflections, calculating crack widths and using finite element analysis programs for slab and wall elements. These calculations become cumbersome when various reinforcement arrangements and deflection behaviour needs to be optimised.

The need for deflection and crack width calculations is well understood. When using FEA programs for the analysis of slabs & walls, it is important to include the contribution of reinforcement in the deflection behaviour. The FEA programs generally accept one value for the thickness and one value for the elastic modulus of their plate elements. Using the two values, these programs predict the deflection and load resistance behaviour of the plate elements. By the use of Inertia-RC, the influence of reinforcement in the section behaviour can be quantified and included in the FEA models by specifying a suitable value of the modulus E for the plate elements.

Method of Use & Data Store

The use of Inertia-RC involves using its two worksheets in conjunction with each other. The worksheet **Inertia-RC** is used for data input and viewing the analysis results. The worksheet "Store" is used for storing data and if required generating input data on a large scale by the use of "copy" and "paste-special" features of Excel.

The use of the two spreadsheets is self explanatory. Input data cells are shaded green. The data input is via changing contents of the green cells.

In the worksheet "Store", data can be stored for as many sections as required, say over 30,000 depending upon the file size. Two buttons in the header of worksheet "Store" allow the addition and deletion of data rows.

Starting from a minimum of 200 sections, data rows are added and or deleted in multiples of 10 and up to 1000 rows at a time. To add more than 1000 rows, the add row step can be repeated.

The worksheet "Inertia-RC" has buttons to allow retrieval of the stored data. Relevant buttons at the top of this worksheet can be clicked to store, retrieve, display-next or display-previous wall data. Click relevant buttons at top of the spreadsheet to store, retrieve, display-next or display-previous retaining wall data.

A data reference number starting from 1001 upwards is used for each section data. This data can be recalled, amended and or re-stored later for any wall by the use of this reference number.

When familiarity has been gained, data in the worksheet "Store" can also be modified directly for ease

The worksheet "Store" also includes columns to show analysis results. When properties are to be calculated for many sections on a large scale, this worksheet can be used independently on its own for data input and simultaneous calculation of section properties.

Analysis Equations for Transformed-Cracked Sections

The equilibrium equation for Neutral axis depth for its various positions relative to the d1 and d2 locations of steels As1 and As2 are listed below.

When both steels As1 and As2 are in the tension zone, the equilibrium equation is given by:

$$e E_c b x/2 + e (x-d1)/x E_s A_{s1} + e (x-d2)/x E_s A_{s2} = 0$$

$$E_c b x^2/2 + (x-d1) E_s A_{s1} + (x-d2) E_s A_{s2} = 0$$

Simplifying it further, the final equation for x becomes:

$$x^2 + 2 (m A_{s1} + m A_{s2})/b x - 2(m A_{s1} d1 + m A_{s2} d2)/b = 0$$

When As1 in compression zone and As2 in tension zone, the above equation becomes:

$$x^2 + 2 [(m-1) A_{s1} + m A_{s2}]/b x - 2[(m-1) A_{s1} d1 + m A_{s2} d2] / b = 0$$

When both As1 and As2 are in the compression zone, the equation for x becomes:

$$x^2 + 2 [(m-1) A_{s1} + m A_{s2}]/b x - 2[(m-1) A_{s1} d1 + (m-1) A_{s2} d2] / b = 0$$

The moment of inertia equations are:

1- when As1=tension As2=tension

$$b*x^3/3 + m * A_{s1}*(x-d1)^2 + m A_{s2}*(x-d2)^2 + m \pi (n1 Dia1^4 + m n2 Dia2^2) / 64$$

2- when As1=compression As2=tension

$$b*x^3/3 + (m-1)* A_{s1}*(x-d1)^2 + m A_{s2}*(x-d2)^2 + m \pi (n1 Dia1^4 + m n2 Dia2^2) / 64$$

3- when As1=compression As2=compression

$$b*x^3/3 + (m-1)* A_{s1}*(x-d1)^2 + (m-1)* A_{s2}*(x-d2)^2 + m \pi (n1 Dia1^4 + n2 Dia2^2) / 64$$

Analysis Equations for Transformed-Uncracked Sections

The area of a transformed section is given by:

$$A_{tr} = b h + (m-1) (A_{s1} + A_{s2})$$

The neutral axis depth (centroid) x from section top edge is given by

$$x = [b h^2/2 + (m-1)(a_{s1}*d1 + a_{s2}*d2)] / A_{tr}$$

The moment of inertia for a transformed section is given by:

$$I_{tr} = b [x^3/3 + (h-x)^3] + (m-1)*A_{s1}*(x-d1)^2 + (m-1)*A_{s2}*(x-d2)^2 + m \pi (n1 Dia1^4 + n2 Dia2^2) / 64$$

Help for Rebar Input Data

When choosing suitable rebar diameters and their spacing or numbers during a design process, it is required to know the areas of rebars being provided in the section. A table is included in the top right of the Inertia-RC worksheet to help for this end.

A snapshot of this table is reproduced here on the right. All green shaded cells in this table can be changed to having any desired value.

Corresponding to the values of bar diameters in the 1st column, the other three columns of the table show the following 3 area values: Area of single rebar, Area of the numbers of each rebar chosen at top of the 3rd column and Area of each rebar in the section width at a spacing chosen at top of the 4th column.

Rebar Area for their Numbers & Spacing			
Width of Section, mm: 1000			
Rebar Dia mm	Area Each mm ²	No of bars 4	Spacing,mm 200
6	28.274	113.097	141.372
8	50.265	201.062	251.327
10	78.540	314.159	392.699
12	113.097	452.389	565.487
16	201.062	804.248	1005.310
20	314.159	1256.637	1570.796
25	490.874	1963.495	2454.369
32	804.248	3216.991	4021.239
40	1256.637	5026.548	6283.185
50	1963.495	7853.982	9817.477

Relative Behaviour of Uncracked and Cracked Transformed Sections

In order to compare the behaviour of Uncracked and Cracked Transformed reinforced concrete sections, the Inertia-RC worksheet includes two tables of analysis results to show the role of each section part. One table shows the behaviour of Uncracked-transformed Section Parts and the other of Cracked-Transformed Section Parts.

A snapshot of these two tables and the data used for the analysis of a section are reproduced below. These two table show the details of equations used and the forces induced in each section part. These tabulated values for each part are strains, stresses, forces and moments in each section part. All of these results correspond to an arbitrarily chosen 0.001 compressive strain at top edge of the section.

Role of Uncracked-Transformed Section Parts Corresponding to 0.001 Compressive Strain at Top Edge					
Section Levels	Equations for Forces & their Location from Neutral Axis x-x	Section Strains	Stress kN/mm ²	Force Value kN	Moment x-x kN mm
Concrete-top	$e E_c b x / 2 @ 2x/3$	0.001	0.031	5,918.2	1,506,479.7
Concrete-bot	$e E_c b (h-x) (x-h)/x / 2 @ 2*(h-x)/2$	-0.001003549	-0.031	-5,960.3	1,522,577.1
As1	$e E_s A_{s1} (x-d1)/x @ (x-d1)$	0.000827145	0.165	665.2	210,094.4
Concrete As1	$-e E_c A_{s1} (x-d1)/x \text{ if } d1 < x @ (x-d1)$	as As1	-0.026	-103.1	-32,564.6
As2	$e E_s A_{s2} (x-d2)/x @ (x-d2)$	-0.000765218	-0.153	-615.4	179,813.5
Concrete As2	$-e E_c A_{s2} (x-d2)/x \text{ if } d2 < x @ (x-d2)$	as As2	0.024	95.4	-27,871.1
Total:				0.0	3,358,529.0

Role of Cracked-Transformed Section Parts Corresponding to 0.001 Compressive Strain at Top Edge					
Section Levels	Equations for Forces & their Location from Neutral Axis x-x	Section Strains	Stress kN/mm ²	Force Value kN	Moment x-x kN mm
Concrete-top	$e E_c b x / 2 @ 2x/3$	0.001	0.031	2,364.1	240,395.2
Concrete-bot	no-force	-0.004015552	0.000	0.0	0.0
As1	$e E_s A_{s1} (x-d1)/x @ (x-d1)$	0.000567286	0.113	456.2	39,476.3
Concrete As1	$-e E_c A_{s1} (x-d1)/x \text{ if } d1 < x @ (x-d1)$	as As1	-0.018	-70.7	-6,118.8
As2	$e E_s A_{s2} (x-d2)/x @ (x-d2)$	-0.003418931	-0.684	-2,749.7	1,433,881.6
Concrete As2	$-e E_c A_{s2} (x-d2)/x \text{ if } d2 < x @ (x-d2)$	as As2	0.000	0.0	0.0
Total:				0.0	1,707,634.3

NB: Concrete-As1 and As2 are areas of concrete displaced by steel areas.

As can be seen from the above two tables, the sum of all forces in the section equate to zero. When the section is uncracked, it resists a moment of 3,358,529.0 kNmm corresponding to 0.001 compressive strain at the top compression edge. When the same section is cracked, the resisted moment reduces to 1,707,634.3 kNmm.

The section size and density values used and the section properties calculated for producing the above two tables are noted below for reference.

Reference: Section Behaviour	
Steel E_s , kN/mm ²	200
Concrete E_c , kN/mm ²	31
Steel Density, kg/m ³	7850
Concrete: b, mm	1000
h, mm	765
Steel: A_{s1} , mm ²	4021.24
As1 Dia, mm	32
Location $d1$, mm	66
Steel: A_{s2} , mm ²	4021.24
As2 Dia, mm	32
Location $d2$, mm	674

Analysis Units	
Force:	kN
Length:	mm
Density:	kg
Volume:	m
Ratio m/mm:	1000

Section Properties			
Property	Concrete	Transformed	Cracked
Area, mm ²	765,000.0	808,844.5	200,391.3
Axis X, mm	382.5	381.8	152.5
Inertia, mm ⁴	37,308,093,750.0	41,367,019,997.7	8,402,383,251.4

By the use of Copy and Paste_Special commands of Excel for the above two table values in a separate workbook, it is possible to calculate analysis results for any other given moment acting on the section. For example, if the analysis results are required for a moment of 800,000 kNm (i.e. 800 kNm), the values obtained/copied from the above two tables need to be multiplied by the following two ratios

$$800000 / 3358528.97409887 = 0.238199523115518 \quad \text{and}$$

$$800000 / 1707634.26664312 = 0.468484391316793 \quad \text{respectively.}$$

Using the Paste_Special_Values_Multiply Command of Excel, the results obtained for the above uncracked and cracked section subjected to a moment of 800,000 kNm become as shown below.

Role of Uncracked-Transformed Section Parts Corresponding to 800,000 kNm moment					
Section Levels	Equations for Forces & their Location from Neutral Axis x-x	Section Strains	Stress kN/mm ²	Force Value kN	Moment x-x kN mm
Concrete-top	$e E_c b x / 2 @ 2x/3$	0.0002382	0.007	1,409.7	358,842.8
Concrete-bot	$e E_c b (h-x) (x-h)/x / 2 @ 2*(h-x)/2$	-0.000239045	-0.007	-1,419.7	362,677.1
As1	$e E_s A_{s1} (x-d1)/x @ (x-d1)$	0.000197025	0.039	158.5	50,044.4
Concrete As1	$-e E_c A_{s1} (x-d1)/x \text{ if } d1 < x @ (x-d1)$	as As1	-0.006	-24.6	-7,756.9
As2	$e E_s A_{s2} (x-d2)/x @ (x-d2)$	-0.000182275	-0.036	-146.6	42,831.5
Concrete As2	$-e E_c A_{s2} (x-d2)/x \text{ if } d2 < x @ (x-d2)$	as As2	0.006	22.7	-6,638.9
Total:				0.0	800,000.0

Role of Cracked-Transformed Section Parts Corresponding to 800,000 kNm Moment					
Section Levels	Equations for Forces & their Location from Neutral Axis x-x	Section Strains	Stress kN/mm ²	Force Value kN	Moment x-x kN mm
Concrete-top	$e E_c b x / 2 @ 2x/3$	0.000468484	0.015	1,107.6	112,621.4
Concrete-bot	no-force	-0.001881223	0.000	0.0	0.0
As1	$e E_s A_{s1} (x-d1)/x @ (x-d1)$	0.000265765	0.053	213.7	18,494.0
Concrete-As1	$-e E_c A_{s1} (x-d1)/x \text{ if } d1 < x @ (x-d1)$	as As1	-0.008	-33.1	-2,866.6
As2	$e E_s A_{s2} (x-d2)/x @ (x-d2)$	-0.001601716	-0.320	-1,288.2	671,751.1
Concrete-As2	$-e E_c A_{s2} (x-d2)/x \text{ if } d2 < x @ (x-d2)$	as As2	0.000	0.0	0.0
Total:				0.0	800,000.0

Units

Inertia-RC works in consistent set of units. Hence any units of Force, Length and Density can be used.

There are 4 units for which their description letters are required to denote them. These letters are used to annotate analysis results with their units.

Typical examples of the 4 units are: Force (kN, lbf), Length (mm, in), Density (kg, lbfm) and Volume (m, ft).

In addition one ratio value is also required for relating size-length unit of the section to its density-length unit for calculating the rebar self weight. For example, if the size-length units is mm and the density-length unit is m, the required ratio value is 1000 = m/mm. Similarly, when the corresponding size-unit is in and the density-length unit ft, the required ratio value is 12 = ft/in.

Limitations

Rebar can be provided at two levels only i.e. As1 and As2 located at distances d1 and d2.

The section shape needs to be rectangular.

The provided reinforcements As1 and As2 are symmetrical about vertical axis of the section.

The influence of axial load (compression or tension) is not included.